

English Summaries

Carlos Andradas

Positive polynomials and polynomial inequalities

The characterization of positive polynomials over the whole affine space \mathbb{R}^n was the goal of Hilbert's 17th problem, solved by Artin in 1929. Then came a host of related collateral problems, concerning properties of the subsets of affine space defined by polynomial inequalities (called semialgebraic sets) and the characterization of the positive polynomials over them. In this paper we offer an exposition of the main results on this topic.

Keywords: positive polynomials, semialgebraic sets, polynomial inequalities, positivstellensatz.

AMS Subject Classification: 14P10, 14P15, 13J30, 12D15.

José Luis Díaz-Barrero i Juan José Egozcue Rubí

Polynomials and reflection coefficients

Polynomials can be represented by their coefficients or by their zeros. The link between these two representations is the Cardan-Viète's formulas that allow expressing coefficients as elementary symmetric functions in the zeros. Backward Levinson's recursion defines reflection coefficients of a polynomial. These coefficients can be used to characterize polynomials. A complete classification of the set of all polynomials is obtained and two theorems on self-inversive polynomials are given. As a consequence of Levinson recursion, a counterpart of Cardan-Viète's formulas is presented. They express polynomial coefficients in terms of its reflection coefficients. Backward Levinson's recursion for polynomials is used again to obtain the characterization of polynomials of Kakeya type by their reflection coefficients. This result leads to an alternative proof of Eneström and Kakeya theorems on the location of zeros of polynomials.

Most of these developments are related to control and signal analysis. In this framework, classical tests for locating zeroes of polynomials are recursive. There are singular cases in which such recursive tests are stopped and perturbation techniques should be applied to proceed. Perturbation techniques, although always successful, are not proven to be well-founded. The

non-singular polynomials are proven to be dense in the set of all polynomials with respect to the L^2 -norm in the unit circle thus giving a mathematical foundation to perturbation techniques.

Keywords: reflection coefficients, location of zeros, Levinson recursion.

AMS Subject Classification: 12D10, 26C05, 30C15.

David Márquez-Carreras i Carles Rovira

Spin glasses

The study of spin glasses initiated from the seventies has had lately great advances in mathematical terms. In this paper we present in an informative way the basic concepts of this area, detailing its mathematical frame, describing the classical models that we consider more important and explaining some of the techniques that are used to study them.

Keywords: spin glasses, Gibbs measure.

AMS Subject Classification: 82B44

Xavier Xarles

The ABC of arithmetics

Some of the latest more important results in number theory, as Fermat last theorem and Catalan's conjecture, have common aspects that unify them. This last years a conjecture has been proposed that explains and generalize these results in a certain sense, the so-called ABC conjecture. In this exposition we present these results and gradually introduce this conjecture.

Keywords: ABC conjecture, Fermat, Mordell, Catalan, radical, diophantine equation.

AMS Subject Classification: 11D61 (11D41, 11G30, 14G05, 14H25).
